Expansion of Mode Shapes and Responses on the Offshore Platform Valdemar

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ABSTRACT

There is a need in the future for maintaining and increasing oil and gas production in the Danish North Sea. Related to this are studies for exploring the potential for extending the lifetime of offshore platforms by implementation of Structural Monitoring Systems (SMS). The project, which this paper is based on, uses an expansion technique as a first step in the sequence of assessing the actual lifetime of a platform. Mode shapes and natural frequencies are estimated using operational modal analysis. The mode shapes are then expanded by expressing each experimental mode shape as an optimal linear combination of selected modes from a finite element model. The offshore platform, Valdemar, which is fully instrumented with accelerometers, GPS, strain gauges and wave radars, is chosen as a case study. Results show that the measured response can be expanded with high precision, which provides valuable information when assessing the actual lifetime of the platform. Also it is shown that the expansion technique can be used for assessment of measurement uncertainties.

KEYWORDS: Offshore Platform, Expansion, GPS, Operational Modal Analysis, Structural Monitoring System.

INTRODUCTION

The Valdemar offshore platform is a Not-Normally-Manned Platform (NNMP) of the type tripod jacket. The platform is operated by Maersk Oil and Gas A/S and was installed in the year 2006. The location of the platform is in the Danish North Sea 250 km of the coast of Denmark. Through a number of oil and gas producing wells in the underground the Valdemar platform distributes untreated well fluids to other platforms for further processing.

The platform is designed to norms and standards with inherit conservatism's for a specific lifetime. The design lifetime typically is governed by accumulated fatigue damage over the years from installation. In the future there is a need for maintaining and increasing the oil and gas production. The potential for extending the original designed lifetime of the platforms can be assessed by installation of an SMS. The purpose of the SMS is to measure and assess the actual lifetime of the platforms [6, 7].

The present study is part of a series of ongoing studies with different participants for development of advanced algorithms and methods for best exploitation of data from SMS measurements ranging from improvement of sensor performance

Fig. 1 – *Valdemar offshore platform.*

and layouts, to post-processing of data such as linear/non-linear system identification, FEM updating, expansion, load identification, wave load calibration and accumulated fatigue monitoring.

INSTRUMENTATION OF THE PLATFORM

The SMS consists of accelerometers, satellite navigation (GNSS stations), strain gauges and wave radars. Four tri-axial servo accelerometers are placed on the topside, with two accelerometers on the cellar deck and two on the roof deck – fig.2a. Two GNSS stations (named GPS sensors) are placed on the roof deck of the topside, and each of them measure the translation of the topside in three directions – fig. 2b. One of the GPS sensors is placed at the same location as one of the accelerometers, giving valuable information about the quality of the signal in different frequency ranges. Two strain gauges are placed on the center column measuring the bending stresses in two perpendicular directions – fig.2c. And finally three wave-radars are placed on the cellar deck, measuring the sea state defined by significant wave height and wave period and estimating the mean sea direction of the waves – fig.2d.

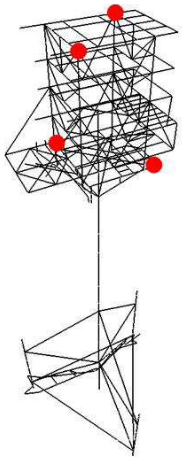

Fig.2a *Placement of accelerometers.*

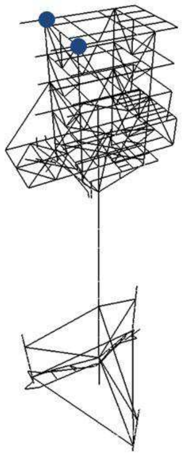

Fig.2b *Placement of GPS sensors.*

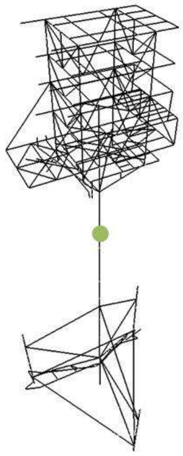

Fig.2c *Placement of strain gauges.*

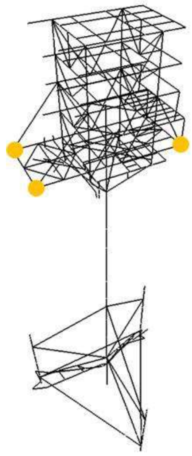

Fig.2d *Placement of wave radars.*

ESTIMATING TRUE AND NOISE SPECTRUM

It is primarily the measurements from the accelerometers and the GPS sensors that are used in the expansion [5]. The measured data from these two measurement systems do not share the same characteristic. In general accelerometers have difficulties measuring in the low frequency range but are superior in the high frequency range. Further the acceleration signal has to be integrated twice to be compatible with the GPS signal, which will lead to added noise in the low frequency area. In general GPS sensors have difficulties measuring in the high frequency range but are superior in the low frequency range. However the GPSs have to measure translations less than a centimeter by a triangulated signal with a baseline of approximately 250 km. This constitutes a challenge as this introduces some noise contributions to the data. Furthermore the GPS signals show sudden outliers in the time domain (clock jumps and/or multipath phenomena) in the present configuration. In this paper the outliers have been removed manually, but the data still have noise contents in the low frequency range.

The theory behind calculating a *true* and a *noise* spectrum is deducted by looking at the correlation between the two signals as a function of frequency [1]. The assumption for using this theory is that the two different measuring devices are calibrated in terms of amplitude, and the noise contaminating the signal is to be considered as white noise.

Consider two signals measuring the same, but contaminated with two different noise sources

$$y_1(t) = s(t) + n_1(t) \qquad \text{Eq.1a}$$

$$y_2(t) = s(t) + n_2(t) \qquad \text{Eq.1b}$$

Where $s(t)$ is the signal and $n_1(t), n_2(t)$ are uncorrelated noise sources with the same properties. Since the noise sources are uncorrelated and also are assumed to be uncorrelated with the signal, the noise spectrum can be found as

$$G_{nn}(f) = G_{y1y1}(f) - G_{ss}(f) \tag{Eq.2}$$

But since the other channel also could be used as a basis for estimating the auto spectrum of the measured output, and since the signal spectrum can be estimated as the absolute value of the cross spectrum between the two outputs, then it is natural to define

$$G_{nn}(f) = \sqrt{G_{y1y1}(f)\ G_{y2y2}(f)} - \left|G_{y1y2}(f)\right| \tag{Eq.3}$$

Which leads to the final expression for the noise spectrum

$$G_{nn}(f) = \left(1 - \gamma_{y1y2}(f)\right)\ G_{yy}(f) \tag{Eq.4}$$

Where

$$G_{yy}(f) = \sqrt{G_{y1y1}(f)\ G_{y2y2}(f)} \tag{Eq.5}$$

And the classical coherence function is defined as

$$\gamma_{y1y2}(f) = \sqrt{\frac{G_{y1y2}(f)\ G_{y2y1}(f)}{G_{y1y1}(f)\ G_{y2y2}(f)}} \tag{Eq.6}$$

Finally the *true* spectrum is defined by

$$G_{ss}(f) = \gamma_{y1y2}(f)\ G_{yy}(f) \tag{Eq.7}$$

The theory is applied to measurements from the accelerometer and GPS placed in the same point of the platform. Fig. 3 shows the spectra from each of the signals as well as the estimated *true* and *noise* signals.

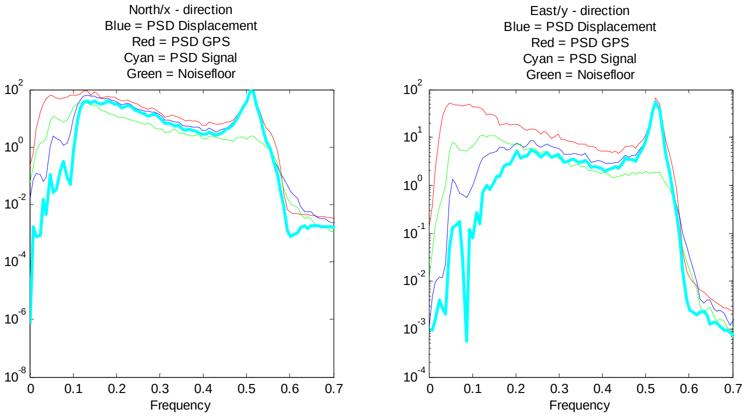

Fig. 3 *Spectra from the GPS and accelerometer placed in the same point on the platform. Left plot: North direction, Right plot: East direction.*

The analysis shows that there is a high correlation between the spectra in the region around the 1st and 2nd natural frequency close to 0.5 Hz. Above 0.7 Hz the accelerometers are superior. It also shows a high correlation in the region 0.15 Hz to 0.4 Hz, where the vibration of the structure is mainly a result of the quasi-static load from the waves. Below 0.15 Hz is a low correlation caused by the noise content in the GPS data. To validate the theory behind the *true* spectra the results have been compared with the spectra from the wave radar, which are consistent in the range of 0.15 Hz to 0.4 Hz. The comparison to wave spectra also reveals that the accelerometers perform well down to around 0.05-0.10 Hz.

The reason for the noise content in the GPS data below 0.15 Hz for this test set-up is due to a combination of a large baseline of 250 km, clock jumps and/or multipath phenomena for the GPS. In addition the present available SMS data is only measured during a summer period with very low measured absolute displacement amplitudes in the range of 0 – 1 mm in the frequency range of 0.05 - 0.15 Hz. Better performances of GPS versus accelerometers are expected for higher displacement levels in this frequency range.

In the frequency range of 0.15 Hz to 0.7 Hz, where there is a high correlation in the spectras, there is an agreement of the measured displacements by the GPS and the accelerometers in the range of 5% to 10% at absolute displacement amplitudes down to 1-2 mm. Rambøll Oil & Gas and others are investigating the possibilities to further improve on the data quality of the GPS sensors.

OPERTIONAL MODAL ANALYSIS

The modal parameters of the structure were extracted using Artemis [2], and with the Stochastic Subspace Identification [3] used as identification algorithm. A total of 5 modes can be detected. Due to uncertainties of the detected modes 4 and 5, these modes are disregarded. Data from the accelerometers was sampled with 128 Hz and an hour of measured data was used when the identification was performed.

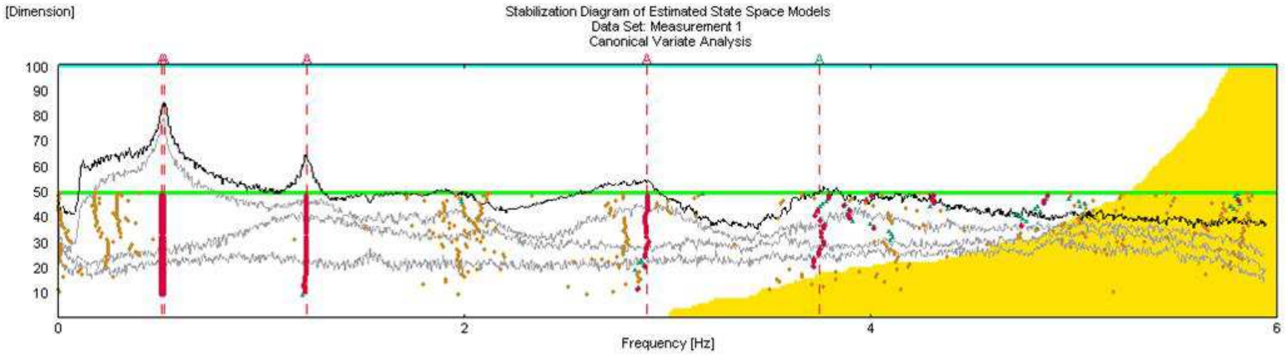

Fig. 4 *Stabilization diagram.*

3 modes were detected. 1st and 2nd modes are the first bending modes in the principal directions, and the 3rd mode is the 1st torsional mode. Modes and frequencies are shown in figure 5a-c.

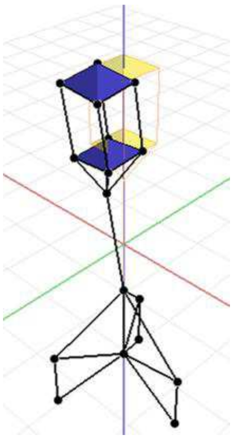

Fig. 5a *1st mode – Frequency: 0.510 Hz*

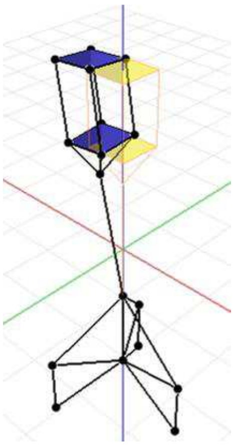

Fig. 5b *2nd mode – Frequency: 0.520 Hz*

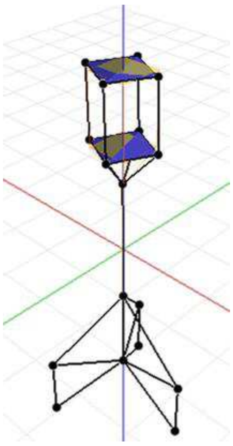

Fig. 5c *3rd mode – Frequency: 1.219 Hz*

EXPANSION

In order to get a better understanding of the dynamic behavior of the structure, both mode shapes and responses from the accelerometers are expanded. The mode shapes are expanded by using an optimized combination of FE modes for each individual experimental mode. This procedure is known as the Local Correspondence (LC) principle [4].

In general it can be shown that the mode shapes of two structures, which are alike, but not equal to each other, can be described as a linear combination of one another. This is often the case when dealing with experimental mode shapes from a structure, and an FE model of the same structure.

The relation between experimental modes and FE modes can be described as:

$$\boldsymbol{A} = \boldsymbol{B}\,\boldsymbol{P} \qquad \text{Eq.8}$$

Where $\boldsymbol{A}$ is the experimental mode shapes extracted using operational modal analysis, $\boldsymbol{B}$ is the FE mode shapes, and $\boldsymbol{P}$ denotes the projection matrix which describes the linear combination.

An estimate of the projection matrix can be found by multiplying the pseudo-inverse of the truncated FE mode shape matrix, with the experimental mode shapes:

$$\widehat{\boldsymbol{P}} = \boldsymbol{B}_{\boldsymbol{a}}^{\dagger}\,\boldsymbol{A} \qquad \text{Eq.9}$$

Where $\widehat{\ }$ indicates an estimate, † the Moore-Penrose pseudo-inverse, and subscript $\boldsymbol{a}$ indicates the truncated version of the FE mode shape containing only coordinates coinciding with where sensors have been placed on the structure.

Once the projection matrix has been found the experimental mode shapes can be expanded by multiplying the full version of the FE mode shapes with the projection matrix:

$$\widehat{\boldsymbol{A}} = \boldsymbol{B}\,\widehat{\boldsymbol{P}} \qquad \text{Eq.10}$$

The expanded mode shapes are shown in figure 6a to 6c.

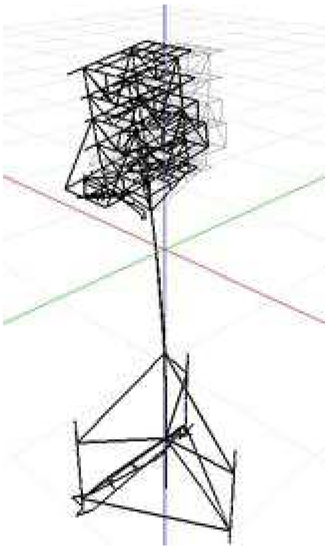

Fig 6a – *1st mode shape, expanded*

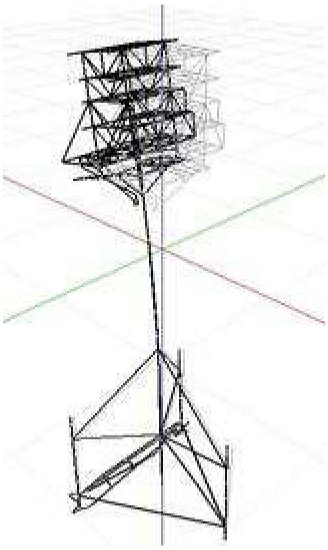

Fig 6b – *2nd mode shape, expanded*

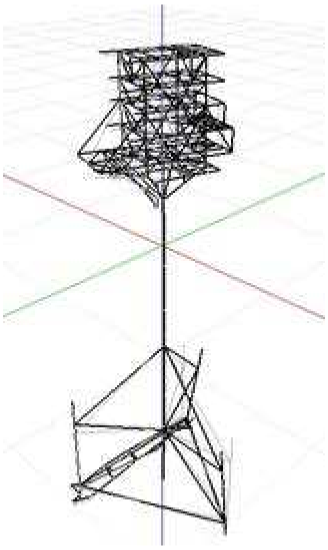

Fig 6c – *3rd mode shape, expanded*

The expansion of the measured response is done through the modal coordinates. An estimate of the modal coordinates can be found by:

$$\widehat{\boldsymbol{q}} = \boldsymbol{A}^{\dagger}\,\boldsymbol{y} \qquad \text{Eq.11}$$

And since an estimate of the expanded mode shapes are found, an expansion of the response can also be found by multiplying the expanded mode shapes with the modal coordinates:

$$\widehat{\boldsymbol{y}} = \widehat{\boldsymbol{A}}\,\widehat{\boldsymbol{q}} \qquad \text{Eq.12}$$

The table below shows the error between the measured and the estimated response from the 4 accelerometers. The z-direction has been discarded since the signal to noise ratio was so low that an expansion didn't make any sense.

The error between measured- and expanded response is found by:

$$\boldsymbol{error}_{\boldsymbol{i}} = \frac{\sigma_{(y_i - \widehat{y_i})}}{\sigma_{y_i}} \qquad \text{Eq.13}$$

σ being the standard deviation.

Channel number:	Error [%]:
1	13.3
2	12.1
3	14.2
4	10.5
5	5.19
6	4.58
7	6.01
8	5.36

Table 1 – *Error measure for each channel.*

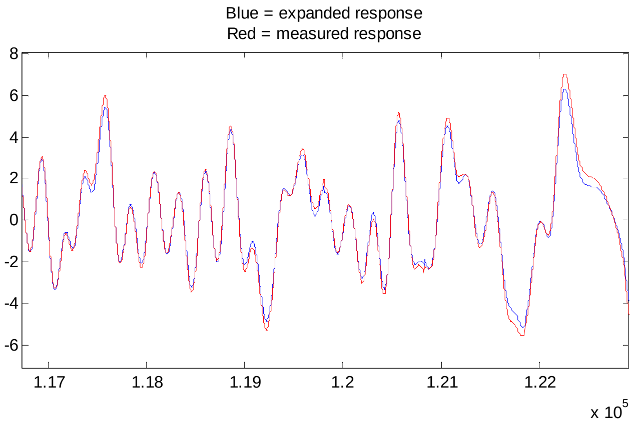

Fig. 7- *Zoom on measured and expanded response with a ~10 % error.*

CONCLUSION

The paper presents the first step in the development of expansion techniques applicable for post-processing of data from a structural monitoring system on an offshore platform. Modal parameters are identified using operational modal analysis, and analysis show that the mode shapes can be expanded with high precision. Furthermore measurement uncertainties are assessed by use of expansion techniques. The studies show that the responses measured by GPS and accelerometers when expanded can be measured with errors in the range of 4% to 14%.

From analysis of signals from two different measurement systems (GPS and accelerometers), it is further shown that it is possible to separate noise from the measurements and creating a *true* signal in the frequency range near DC, by looking at the correlation between signals from a GPS and an accelerometer placed in the same position.

The separation of noise from the measurements and the estimation of the errors for the expanded responses are of great importance for the future assessment of the actual lifetime of the platform.

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